These facts have been ascertained for each of the 73 centres examined. It is obviously impossible here to indicate even the general conclusions thus arrived at, owing to the large number of separate observations which cannot be briefly collated. Reference must therefore be directed to the original paper.

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"The Influence of Stress and Strain on the Physical Properties of Matter. Part I. Elasticity—*continued*. The Velocity of Sound in Metals and a Comparison of their Moduli of Longitudinal and Torsional Elasticities as determined by Statical and Kinetical Methods."

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[PLATE 2.]

We owe to Wertheim* a series of carefully executed experiments on the longitudinal elasticity of metals both by statical extension and by longitudinal and transverse vibrations. From these researches it would appear that the values of the moduli of longitudinal elasticity as determined for several metals by the first of these three methods are, as might be expected, less than those obtained by the other two. The differences, however, are very much greater than can be accounted for by the heating and cooling effects of contraction and elongation, and the author has already pointed out what he believes to have been in a great measure the cause of these discrepancies.† As a few observations made with two or three different metals had seemed to him to show the possibility of obtaining more concordant results, he was encouraged to extend his investigations to other metals, and moreover to institute a comparison between the values of torsional elasticity which could be obtained by statical and kinetical methods. It had originally been the author's intention to use the same specimens of the various metals as were employed in his previous experiments on moduli of elasticity and electrical conductivity,‡ but on applying to Messrs. Johnson, Matthey and Co. to have these specimens fused and redrawn, he was informed that what was desired would be almost if not quite impossible, inasmuch as several of the metals if fused in small quantities would be rendered too brittle for the

* 'Annales de Chimie,' vol. 12, 1844.
† 'Phil. Trans.,' vol. 174, 1883 (Part 1), p. 14.
‡ *Loc. cit.*

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process of wire drawing. Accordingly by request, Messrs. Johnson and Matthey with their usual courtesy prepared specimens of platinum, silver, copper, aluminium, lead, platinum-silver and German silver in the same manner and of the same degree of purity as before. The wire-drawers had received special instructions to avoid kinks and to secure uniformity in the diameters of the wires throughout their lengths. Experiment I, which may be taken as representative of the degree of uniformity obtained in the diameter of the various wires, shows that in this last respect the instructions had been well carried out; nor could any kinks be detected in the wires. The results given in Experiment I were obtained by means of a gauge reading to $\frac{1}{100}$th of a millimetre; by estimation it was easy to measure to $\frac{1}{1000}$th of a millimetre.

Experiment I.

Distance in feet from one end of the wire at which the gauge was applied.	Gauge-reading in centimetres.
1·5	0·1088
3·0	0·1098
4·5	0·1087
6·0	0·1091
7·5	0·1092
9·0	0·1089
10·5	0·1091
12·0	0·1088
13·5	0·1093
15·0	0·1096
16·5	0·1089
18·0	0·1091
19·5	0·1091
21·0	0·1091
22·5	0·1092
24·0	0·1092
25·5	0·1088
27·0	0·1092
28·5	0·1093
30 0	0·1093

The mean value of the gauge-readings is 0·10918, and in no case does a gauge-reading differ from this mean by more than $\frac{1}{2}$ per cent. After the diameter had been determined for each of the wires in a similar manner by means of the gauge, they were made into coils of more than one foot diameter, and the diameter again determined from the apparent loss of mass in water at 4° C. and from the length. The values for the diameters obtained by the last method agreed very closely with those got by means of the gauge.

The Longitudinal Elasticity as determined by the Method of Statical Extension.

The mode of experimenting by statical extension has already been described,* and the precautions which were used then were used now; but in these fresh trials the author availed himself of a device whereby the departure from "Hooke's law," which had been formerly observed more or less with all the wires, can be done away with. This device the author owes to a perusal of the investigations of Professor G. Wiedemann on statical torsion.† Wiedemann has proved that though on first applying the loads used for twisting the wire the torsional strain increases in greater proportion than the stress, the frequent repetition of these loads gradually diminishes this want of proportionality. He has further shown that the process may be much facilitated by repeatedly putting the wire into torsional oscillations whilst under the influence of the torsional stress. In a similar manner the author now found that if the wire when under the influence of a load causing longitudinal stress were set oscillating longitudinally, by alternately pressing with the hand on the scale-pan and removing the pressure, the range for which "Hooke's law" held good was sensibly increased,‡ so that he was able to use larger loads than could otherwise have been used without passing beyond the boundaries of perfect elasticity. The following experiment will serve to show the degree of accuracy attainable.

* 'Phil. Trans.,' vol. 174, 1883 (Part I), pp. 3, 4.

† 'Phil. Mag.,' Jan. and Feb., 1880.

‡ It should be mentioned here that in all cases it is advisable to allow a rest after oscillating the wire in this way for the first time, and afterwards to oscillate it again just before testing.

Experiment II.—Platinum-silver.

Number of series of experiments.	Load on the wire in kilos., P.	Difference between consecutive loads, ΔP.	Scale-reading in half-millimetres, S.	Difference between consecutive scale-readings ΔS.	ΔS/ΔP.	Mean values of ΔS/ΔP in each series.
I	10	..	21·78	..	..	1·1958
	16	6	14·57	7·21	1·2015	
	12	4	19·35	4·78	1·1950	
	16	4	14·58	4·77	1·1925	
	14	2	16·93	2·35	1·1750	
	12	2	19·36	2·43	1·2150	
II	10	..	21·64	..	..	1·2017
	16	6	14·40	7·24	1·2067	
	10	6	21·63	7·23	1·2050	
	*12	..	19·19	..	..	
	14	2	16·77	2·42	1·2100	
	16	2	14·40	2·37	1·1850	
III	*10	..	19·63	..	..	1·1950
	14	4	14·86	4·77	1·1925	
	18	4	10·07	4·79	1·1975	
	10	8	19·63	9·56	1·1950	
	14	4	14·78	4·85	1·2125	
	18	4	10·06	4·72	1·1800	
	10	8	19·60	9·54	1·1925	
IV	10	..	19·61	..	..	1·1950
	14	4	14·79	4·82	1·2050	
	*14	..	11·49	..	..	
	18	4	6·72	4·77	1·1925	
	14	4	11·45	4·73	1·1825	
	*18	..	16·86	..	..	
	14	4	21·59	4·73	1·1825	
	10	4	26·44	4·85	1·2125	

* Signifies that the scale was shifted, either by slightly increasing or diminishing the permanent load as in the first two cases, or by unclamping the scale and vernier and entirely readjusting, but so that the same length of wire might be under examination as before.

The mean value of $\Delta S/\Delta P$ for the four series is 1·1969 and the probable error is 0·10 per cent. The values of $\Delta S/\Delta P$ obtained with the temporary stresses of 2 kilos., 4 kilos., 6 kilos., and 8 kilos. are, within the limits of errors of observation, equal, and moreover are independent of the permanent load on the wire.

The results recorded above were not got without the exercise of very great precaution both in adjusting the scale and vernier, and in preventing either the wire under examination or the comparison-wire from twisting after the adjustment had been completed. Should such twisting take place the vernier will not move with sufficient freedom up and down the scale.* Indeed in this experiment there are here and there slight traces of the vernier sticking in the scale, and in most of the other experiments it was considered advisable to unclamp both scale and vernier and completely readjust them, so that slightly different lengths of the wire might be under examination.† The next experiment furnishes an example of this mode of treatment.

Experiment III.—Piano-steel.

Temporary load, 8 kilos.; permanent load, 10 kilos.

Length of the wire under examination in centimetres, l.	Temporary alteration of length in centimetres, ΔS.	$\Delta S/l$.
753·8	0·5245	0·0006955
750·8	0·5195	0·0006920
754·0	0·5233	0·0006940

The mean of the numbers in the third column is 0·0006938, with a probable error of 0·09 per cent. It is evident therefore that with care considerable accuracy can be obtained. The elasticity of piano-steel within the limits of the loading here employed had been previously proved to be quite perfect, provided the precautions adopted in this and the other experiments were taken, so that it was considered unnecessary to try the effect of lesser loads, but with all the other metals at least two loads were used, and for these loads Hooke's law, "*ut extensio sic vis*," could by proper treatment of the wire be made to hold good within the limits of errors of observation.

* The author may mention here that he found it to be a great convenience to place four blocks of well-planed wood on the ground close to the four sides of the scale-pan, and to anchor the bar fastened to the comparison-wire by fine wires attached to the ends of the former and secured in a horizontal position.

† The different lengths were not measured until the series of temporary alterations of length produced by the various changes of load was completed, so as to avoid prejudice on the part of the observer.

The Longitudinal Elasticity as determined by the Method of Longitudinal Vibrations.

Preliminary Trials.

As it was found necessary to use the syren more or less throughout this part of the investigation, some experiments were made with a view of ascertaining how far this instrument could be relied on for determining the number of vibrations made in a given time. First, attention was directed to the registering apparatus, with the object of determining how far the putting of this in, or the taking of it out of action, would introduce error. Any error of the kind would be more manifest when the number of vibrations was determined for a short space of time than for a long one, and in the following experiment it will be seen that there is no appreciable error due to the inertia of the register.

Experiment IV.

The number of vibrations recorded in one minute by the syren in each of three trials, when the syren was kept in unison with a monochord, were:—

Number of trial.	Number of vibrations.
1	813 × 20
2	812 × 20
3	811 × 20
Mean.....	812 × 20

The number of vibrations recorded in five minutes was next ascertained by one trial to be 4064 × 20, which would give 812·8 × 20 per minute, or only 0·1 per cent. higher than before. The pitch of the monochord was then altered, and several fresh experiments similar to the above ended in showing that sometimes the number of vibrations per minute registered in the shorter-timed trials appeared to be greater, and sometimes less, than the number registered in the longer-timed trials, the difference being in all cases equally slight with the above.

Again, the syren was employed to determine the pitch of several of Koenig's forks; thus in a single trial, in each case of three minutes' duration, the instrument registered 256·7 and 513·8 vibrations per second respectively for two forks, which were marked with the numbers 256 and 512. Several other trials of similar accuracy, sometimes giving slightly greater, and sometimes slightly less vibration-fre-

quencies than those marked on the forks, rendered it evident that it was possible to determine with great accuracy the vibration-number of a tuning-fork by means of the syren, when the notes of these two instruments are compared *directly* with one another.* The case, however, was different when a monochord was tuned to the note of a fork, and the former then compared with the syren. Thus the monochord was tuned to unison with a Koenig's fork marked 384, and afterwards the vibration-frequency of the former was determined by the syren. Two trials resulted in giving the number of vibrations per second as 387 and 386. In a similar manner, using a fork marked 512, six trials with the syren gave the vibration-frequency as follows :—515·8, 516·8, 514·3, 515·8, 516·8, 515·4, with a mean value of 515·8. The experiments with both forks therefore gave values for the vibration-frequencies which were about $\frac{3}{4}$ per cent. too high, and yet the monochord after these trials was in each case still in perfect unison with the fork with which it had been compared. As the assistant (Mr. Furse) seemed to think that the error arose from drawing the bow too strongly across the wire of the monochord, in the endeavour to make the sound of this instrument of sufficient intensity to be heard at the same time as that of the syren, a screen was placed so as partly to shield the sound of the latter from the manipulator of the monochord, and this plan proved to be successful, for now the syren recorded the same vibration-frequency for both monochord and tuning-fork, and several experiments of the same kind with forks of different pitch manifested that with the precaution mentioned above the syren could be made to determine the pitch of the monochord as accurately as the pitch of the tuning-fork.

Matters having been so far satisfactorily arranged, a considerable number of trials were made, for the purpose of deciding on the best mode of arranging the wire which it was desired to throw into longitudinal vibrations. In the first instance the wire to be examined was stretched horizontally, and clamped at one end to a block of iron secured to a window-sill. Towards its other extremity the wire passed over a fixed pulley, so that by placing weights on a scale-pan attached to this extremity any required degree of stress could be put upon the wire. Before going over the pulley the wire passed through the jaws of a strong vice, so that when stretched sufficiently it could be firmly clamped by means of the latter, and was protected from injury by placing pieces of hard wood between it and the jaws of the vice. Both the vice and pulley were firmly clamped to a very stout table. When fixed for examination the wire was thrown into longi-

* The author has again to thank Mr. Furse, the curator of the Museum of King George III, at King's College, for his assistance; as also Mr. H. A. Reatchlous, one of the students of the Physical Laboratory. Both these assistants are musicians, and the former is especially remarkable for his skill in manipulating the syren.

tudinal vibrations by rubbing it as lightly as possible with a resined glove; a monochord was tuned in unison with the wire, and the pitch of the former then determined by means of the syren. In this way copper, platinum, silver, and platinum-silver were examined, each with two different lengths, one length being about half that of the other. In all cases the shorter length gave a greater number of vibrations in proportion to the inverse of the length than the longer one. The average extent of the deviation is exhibited in Experiment V.

Experiment V.—Platinum Wire.

Length of the wire examined in centimetres, l.	Number of vibrations per second, n.	$l \times n$.
967·4	146·47	141,700
495·7	186·90	142,230

Here in the case of the shorter length the product $l \times n$ is about 0·4 per cent. higher than the same product for the longer length. As much greater accuracy than this was to be aimed at for the purpose in view, several days were spent in endeavouring to ascertain what flaws there might be in this mode of experimenting. In the first place the permanent loads placed on the scale-pan were gradually increased in amount, and as the wires were hard drawn, this could be done to a considerable extent without causing any sensible permanent elongation. Evidently, however, the source of error was not to be detected in this way, for though a slight change in the pitch of the note could be detected when the note was reduced below a certain comparatively small amount, yet after this amount of stress had been exceeded, no further addition of load seemed to produce any appreciable effect. Secondly, the clamp and vice were shifted and more firmly secured, but still with no good result.

After this it was decided to place the wire vertically, and as the room was very lofty a considerable length could be tested in this new position. The rest of the arrangement was the same as before, except that now the pulley was dispensed with and the wire was clamped at its upper extremity to a very massive iron plate, and hung freely down through the jaws of the vice before clamping with the latter. The following table gives the mean values of the products $l \times n$ for the different wires examined in vertical and horizontal positions, where l and n have the same signification as before:—

Table I.

Metal.	$l \times n$, position vertical.	$l \times n$, position horizontal.
Copper	192,650	195,567
Platinum	136,410	141,700
Silver	136,410	140,067
Platinum-silver	140,500	142,200
Aluminium	243,900	
Piano-steel	261,700	

The results shown in this table are far from being satisfactory, and the numbers in the third column differ from those in the second column by amounts which are in all cases considerably larger than the differences in the values of $l \times n$, as determined for different lengths of the same metal, either in the vertical or the horizontal position.

Final Method of Experimenting.

Though the notes obtained by rubbing the wires longitudinally were fairly clear and well defined when the wire was arranged according to either of the above methods, yet the divergence of the results given above was such as to induce the author to try a third method, which, when certain corrections have been applied for want of rigidity of the masses to which the ends of the wire are clamped, seems to be capable of considerable accuracy. In fig. 1, Plate 2, AB is a hollow box,* made of wooden planks half an inch thick. The length of the box is 600 cm., the breadth 10 cm., and the depth 10 cm. At one end of the box is a pulley C, round which the wire passes to the scale-pan S. D and E are two pairs of stout blocks of wood, each of which can be firmly clamped to any part of the box by a pair of very stout wooden screws.† The stout blocks of wood carry each an iron clamp, by which the wire can be secured, and the blocks of wood together with the screws are all well insulated from the box by means of thick layers of baize M, so that the vibrations of the wire cannot be imparted to the box.‡ The wire is first clamped to the wooden

* There is no advantage in having a *hollow* box; a solid piece of wood or metal of sufficient stoutness would answer the purpose equally well.

† Only one of each pair of screws is shown in the figure.

‡ This the author found to be a matter of some considerable importance, as the note was very much clearer and purer when the blocks were insulated from the box than when they were not. A rather curious case of synchronism occurred in one of the earlier experiments where the blocks were not insulated from the box. An iron wire had been arranged to give, as far as could be judged by the note, 512 vibrations per second. The note was, however, very far from clear; but by shortening or

block at D, and is then passed over the pulley at C, so as to be stretched to any required extent by placing weights on the scale-pan S. When the required stress is attained the wire is clamped at E, and the part to the right of E having been detached from the pulley and scale-pan, is drawn on one side and rested on some non-conductor of sound, such as baize or flannel. In several cases, before the last-mentioned adjustment had been completed, the blocks of wood at E were shifted backwards or forwards, until the note given out by the longitudinally rubbed wire was in unison with a Koenig's tuning-fork, but in others the pitch of the note was determined with the syren. If the clamps at D and E had been secured to perfectly rigid supports, the number of vibrations obtained when the wire was clipped in the centre would have been exactly double the number when the wire was free, except at both ends, but in consequence of lack of rigidity of the supports at D and E, the note given out in the former case had *less* than double the number of vibrations of the note in the latter. Now Lord Rayleigh has proved for transverse vibrations* that when, as in the present instance, the mass at each end is large compared with the force of the spring which urges the extremity attached to the mass towards the position of equilibrium, any slight yielding of the supports will cause a *rise* in pitch, and will produce the same effect as if the wire had been shortened in the ratio of $1 : 1-k/n^2$, where k is a constant, if we experiment with the same length of the same wire under the same conditions as regards the nature of the supports, and n is the tone of the wire. Lord Rayleigh's mathematical reasoning can be equally applied to *longitudinal* vibrations, and it is obvious that by obtaining the number of vibrations of the wire when free, except at both ends, and then when clipped in the centre, we may determine the amount by which the yielding of the supports heightens the pitch of the note. For the sake of greater accuracy the number of vibrations yielded when the wire was clipped one-third of its whole length from one end was in some cases also ascertained. The next experiments will sufficiently illustrate the mode of proceeding.

lengthening the wire slightly it was very much improved, though of course of a different pitch. The very marked want of clearness was presently found to arise from synchronism between the time taken by a pulse to pass from one block to the other *through the wood* and the time taken to pass from end to end of the wire and back again. When the blocks were insulated from the box the want of clearness vanished and the pitch of the note rose 6 or 7 per cent.

* 'Theory of Sound,' vol. 1, § 135.

Experiment VI.—Platinum-silver Wire.

Number of vibrations per second.*	Remarks.
259·7 × 1	Wire free except at both ends.
254·7 × 2	Wire clipped in the centre.
253·9 × 3	Wire clipped at a point one-third of the whole length from one end.

Let k be the equivalent shortening of the wire when it is free except at the two ends; then since the velocity of sound along the wire will be the same in all three cases we must have:—

$$\left(1-\frac{k}{1^2}\right)259{\cdot}7 = \left(1-\frac{k}{2^2}\right)254{\cdot}7; \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\left(1-\frac{k}{1^2}\right)259{\cdot}7 = \left(1-\frac{k}{3^2}\right)253{\cdot}9; \quad . \quad . \quad . \quad . \quad (2)$$

From (1) we obtain $k = 0{\cdot}0255$,

and from (2) $k = 0{\cdot}0250$.

The mean of these two values of k is 0·02525, and since the length of the wire examined was 553·85 cm., the velocity of sound in centimetres per second obtained from the three sets of trials given in Experiment VI will be 280,440, 280,380, and 280,500 respectively, with a mean of 280,440 and a probable error of only 0·008 per cent.

Experiment VII.—Silver Wire.

Length, 553·85 cm.

Number of vibrations per second.	Remarks.
253·8 × 1	Wire free except at both ends.
253·1 × 2	Wire clipped in the centre.
253·0 × 3	Wire clipped at a point one-third of the whole length from one end.

Adopting the same mode of procedure as before we obtain for k the two values 0·00366 and 0·00354 with a mean of 0·00360. Thus the

* The numbers given under this heading are in this and the next experiments the mean values resulting from several closely accordant trials.

velocity of sound in centimetres per second as deduced from the numbers in the first column will be, within 0·001 per cent., in all three series of trials the same, namely, 280,100. Experiments VI and VII furnish results which are rather more consistent with each other and attended with a slightly less probable error, as judged by the departure of each individual value of the velocity of sound from the mean value, but still the agreement seemed to be very good in the case of the other metals. Thus with annealed iron wire of the same length as the silver wire, and which was tested not only for the fundamental note and the first and second octaves but also for the third octave, the following values of the velocity of sound were deduced:—

Velocity of sound in centimetres per second.	Note.
509,700	Fundamental.
510,000	First octave.
510,400	Second octave.
508,400	Third octave.
509,600 mean.	

Again a hard drawn copper wire of the same length gave the following results:—

Velocity of sound in centimetres per second.	Note.
395,600	Fundamental.
395,400	First octave.
396,400	Second octave.
395,800 mean.	

The velocity of sound for the other wires given in the next table was determined only from observation of the fundamental note and the first octave. All the metals, except the piano-steel, the annealed iron and the German silver, were obtained from Messrs. Johnson, Matthey and Co., and were stated to be chemically pure.

Table II.

Metal.	Condition.	Density.	Velocity of sound in metres per second.
Piano-steel	Unannealed	7·7475	5198
Iron.....	Annealed	7·6831	5096
Copper	Unannealed	8·8976	3958
German silver	,,	8·6320	3860
Platinum-silver	,,	12·1900	2804
Silver.....	,,	10·4668	2801
Platinum.....	,,	21·0500	2750

In the next table will be found a comparison between the moduli of longitudinal elasticity as obtained by the statical and kinetical methods for steel, copper, platinum, platinum-silver, and silver. All the results given in the table were obtained with as much care as those already quoted.

Table III.

Metal.	Young's modulus in grams per square centimetre as obtained by the kinetical method. e_k.	Ditto as obtained by the statical method. e_s.	Ditto supposing no heat to be gained or lost during the testing. e'_s.	$\frac{e_k - e'_s}{e_k}$.
Piano-steel	2133 $\times 10^6$	2140 $\times 10^6$	2144 $\times 10^6$	−0·0051
Copper	1316	1323	1326	−0·0076
Platinum	1622	1623	1625	−0·0018
Platinum-silver	997	1001	1004	−0·0070
Silver.....	835·6	828·6	831·1	+0·0054

It will be seen from Table II that the values of Young's modulus, as determined by the statical and kinetical methods, agree with each other within less than 1 per cent., and that on the whole the values obtained by the *former* of the two methods are slightly *greater* than those obtained by the *latter* method.*

* The author is inclined to attribute this to the fact that with hard-drawn metals loading always produces a slight amount of temporary twisting. This (see Experiment II) tends to produce a slight degree of sticking between the scale and vernier used in the statical method. With well annealed wires this temporary twisting does not result from loading.

The velocity of sound as calculated from the values of e'_s and the density would in no case differ from those obtained directly by so much as $\frac{1}{2}$ per cent.

The Velocity of Sound is Independent of the Temporary Load.

Experiment VIII.

A well annealed copper wire about 1 mm. in diameter was weighted with 18 kilos. and left with this load on for some time. It was then tested with various loads up to 18 kilos. with the following results.

Load on the wire before clamping.	Number of longitudinal vibrations in three minutes.*
6 kilos.	4048·5 × 20
10 „	4056·3
14 „	4054·9
18 „	4057·0
Mean of last three....	4056·1

It is evident from the above that except for the lightest load, the velocity of sound is entirely independent of the temporary load, and even with the load of 6 kilos. there is not a departure from the mean value obtained with the other three loads of more than 0·2 per cent.†

Similarly with a pianoforte steel wire of 0·08 cm. diameter, not the slightest difference in the pitch of the note could be discerned with loads which varied from 16 kilos. up to 30 kilos.

The temporary alteration of density which was produced in either of these two experiments would be too small to cause of itself a perceptible alteration of pitch. Thus in the case of the piano-steel wire, the temporary change of density resulting from the increase of the load from 16 kilos. to 30 kilos. would only cause a change in the pitch of the note of 0·03 per cent.

The Effect of Permanent Extension on the Velocity of Sound.

The author has shown‡ that when an iron wire has recently suffered permanent extension the longitudinal elasticity as determined by the

* The numbers in this column are the means of several observations with each load.

† The note with the load of 6 kilos. was not quite so clear as that obtained with the other three loads.

‡ 'Phil. Trans.,' vol. 174 (Part I), 1883.

statical method is decidedly less than when rest has been allowed after the permanent extension. The effect of rest in increasing the elasticity appeared to be less the less the temporary load used in testing the elasticity, and it seemed of some interest to ascertain in the first place the effect of permanent extension on the velocity of sound, and in the second whether rest would appreciably alter the velocity. The following experiment was therefore tried.

Experiment IX.

A well-annealed iron wire was stretched sufficiently by a temporary load to give a clear note when rubbed longitudinally. The pitch of the note was then taken on a monochord, and again after the wire had suffered more and more permanent extension, the same temporary load being used throughout, and the wire being shortened to its original length after each permanent extension.

Percentage permanent increase of length.	Length in centimetres of the wire of the monochord when the latter gave the same note as the longitudinally rubbed wire.
0·00	29·34
1·82	29·34
4·00	29·34
6·36	29·34
9·10	29·34
13·64	29·34
17·00	29·34

It will be seen from the above that the pitch of the note remained, as far as could be judged, absolutely unaltered by the permanent extension. After one hour's rest, however, it seemed to be appreciably sharper, the frequency as determined by the syren being 458·5 as against 458·1, the frequency before stretching and before rest, though after recent permanent extension. It would thus appear that rest after permanent extension does very slightly increase the velocity of sound in the case of iron.

The permanent increase of velocity of sound can, however, be almost, if not entirely, accounted for by the diminution of density. The latter amounted altogether to 0·17 per cent., and this diminution would cause an increase in the velocity of sound of 0·085 per cent., whereas the actual increase observed was only 0·087 per cent. We must therefore conclude that the *elasticity* was not appreciably affected permanently by the permanent extension, though rest did

apparently very slightly increase the elasticity after recent permanent extension.

Several other experiments of a like nature were made with annealed iron, but all seemed to show that the elasticity as tested by the method of longitudinal vibrations was not appreciably altered permanently by permanent extension.

Similar experiments to the above were made with annealed copper wire, and with similar results, except as regards the effect of rest, which in this case produced no change.

A Comparison of Moduli of Torsional Elasticity as determined by the Statical and Kinetical Methods.

Statical Method.

The wire to be tested, about 28 feet in length, was fastened at its upper extremity to a clamp secured to a stout iron bracket. The lower extremity of the wire was clamped at C, fig. 2, Plate 2, the extremity of a brass rod $\frac{1}{2}$ inch in diameter, and $1\frac{1}{2}$ feet in length; the rod passed vertically through the centre of a horizontal brass plate P, 8 inches in diameter, so that half of the rod was above and the other half below the plate; the lower half of the rod terminated in a hook to which was suspended a scale-pan, S, weighing 2 kilos. The torsion was effected by placing weights* in two scale-pans, T, made of cardboard, and weighing 10 grams each; the scale-pans, T, were fastened each to one extremity of a light silk thread, which passed over a pulley, W, and was wrapped a few times round the rod; the two threads were wrapped round the rod in opposite directions, so that when equal weights were placed in the scale-pans, T, a torsional couple was produced. The plate P was divided at its circumference into degrees, and by using a fine steel pointer (fig. 3), placed above the rim of the plate, and nearly but not quite touching it, it was possible to estimate to one-tenth of a degree. The stands carrying the pullies, W, were capable of being moved either vertically or horizontally in any direction, and great care was taken to ensure that the parts of the silk threads between the rod and the pullies were parallel to each other. In order to accomplish this last the adjustments were made in the first instance as nearly correct as the eye could judge; torsion was now imparted in the manner described, and after the plate had been twisted through about 360 degrees, the position of the end of the pointer, as regards its distance from the rim of the circular plate, was noted. If this distance was not the same as before the application of the torsion, one or both of the pulley stands were shifted in a horizontal plane until there was no perceptible difference in the distance

* These weights were of thin strips of German silver and were each made very accurately equal to 10 grams.

of the end of the pointer from the rim of the plate before and after torsion.

When the horizontal adjustment was satisfactorily completed, the silk threads could be made perpendicular to the axis of the rod by shifting the pulley stands up or down in a vertical plane, until any given load in each pan produced a maximum twisting effect. The pullies, W, were large and light, and so delicately balanced that the loss from friction was very slight. This loss from friction, small as it was, could be eliminated in the following manner:—Suppose that the torsional stress has twisted the plate through a certain number of degrees, the plate is now twisted very carefully by hand a little further, and this stress then very gradually relaxed; let D_1 be the present position of the pointer. Again, let the original torsional stress be carefully relaxed a little, and then very gradually restored, the pointer will now take up a new position, D_2. The true position of the pointer, if there were no friction, would be $D_1 + D_2/2$. At least ten trials were made with each of the torsional stresses employed, and the mean of the different readings, which accorded very well with each other, was taken.

In order to apply the torsional stress with sufficient gentleness, the following plan was adopted:—Two smooth blocks of wood were placed with a face of each in contact with two opposite sides of the scale-pan, S, so that neither the pan nor the plate, P, would move when weights were put in the smaller pans, T. As soon as the latter were loaded, the blocks of wood were gradually and gently removed from the sides, so as to permit of the torsional stress producing its effect by slow degrees.*

Prof. G. Wiedemann has already shown in his experiments on torsion by the statical method, that the torsional elasticity is independent of the amount of longitudinal stress which may be acting on the wire at the same time as the torsional stress,† and the author has also proved that this is the case when the kinetical method is adopted.‡ However, it was considered advisable to make a few preliminary experiments, to ascertain whether temporary loading would affect the torsional rigidity. It is unnecessary to enter into the results of these preliminary trials further than to say that they fully verified Prof. G. Wiedemann's previous observations for all the metals which were examined.

Before commencing the actual testing the wire was frequently set

* This plan was found to answer the purpose very well. Professor G. Wiedemann has adopted a much more elaborate arrangement for effecting the same object, but the author cannot help thinking that with care such a device as that mentioned above is quite sufficient to prevent the stress from being applied too suddenly.

† 'Wiedemann's Annalen,' vol. 6, 1879; 'Phil. Mag.,' Jan. and Feb., 1880.

‡ 'Phil. T a ıs.,' vol. 177 (Part II), 1886.

in torsional oscillation with a load in the pan, S, slightly greater than any which it was intended to use, and after each set of oscillations a long rest was allowed. The object of this preliminary treatment* was to extend the limit of elasticity as much as possible.

Again, immediately before the testing the wire was set in torsional oscillation, but not through a greater arc than that through which it was intended eventually to twist the wire.† Finally, the loads which were intended to be used in the pans, T, were put in and taken out ten or a dozen times, and then the actual trials began, the load in the pan, S, having been some time previously reduced to the amount to be used in the trial. The following experiment will give a fair notion of the degree of accuracy which was obtained.

Experiment X.

A hard-drawn aluminium wire, about 800 cm. long, and 0·1 cm. in diameter. The load on the wire was merely that of the scale-pan, *i.e.*, 2 kilos.

Load in each pan producing torsion.	Position of index.	Degrees of torsion produced by the torsional stress.
0 grms.	230·70	—
20 „	51·70	179·00
0 „	233·93	182·23
20 „	50·95	182·98
0 „	231·20	180·25
20 „	51·70	179·50
0 „	233·90	182·20
20 „	52·80	181·10
0 „	233·05	180·25
20 „	51·73	181·32
0 „	233·05	181·32
	Mean.....	181·02

The probable error of the mean value 181·02, given above, is 0·14 per cent.

A set of observations was next made with 10 grams instead of 20 in the pans, T, the mean number of degrees of torsion being in this case 90·43, with a probable error of 0·36 per cent. Within the limits of probable error 90·43 is the half of 181·02. In calculating

* The period of this treatment depends upon the nature of the metal; with iron it is advisable that it should extend over a couple of days.

† The object of this will be seen from the author's paper on "The Internal Friction of Metals," 'Phil. Trans.,' vol. 177 (Part II), 1886.

this result it was assumed that the number of degrees of torsion produced with 10 grams in each of the pans, T, was $\frac{90{\cdot}43 + \frac{1}{5}(181{\cdot}02)}{2}$, or 90·47.

The value of the modulus of torsional elasticity in grams per square centimetre can be found from the formula—

$$r_s = \frac{L \times D \times P \times 360}{n \times S^2},$$

where L is the length of the wire in centimetres, S the section in square centimetres, P the number of grams in each pan, n the number of degrees of torsion, r_s the modulus of torsional elasticity determined by the statical method, and D is the arm of the couple P × D in centimetres. The value of D was determined very carefully by a wire gauge reading to $\frac{1}{100}$th of a millimetre, due allowance being made for the thickness of the silk thread, and proved to be 0·9668 cm. The value of L varied in the different experiments from 650 to 800 cm., and within the limits of errors of observations the strain, as in the above experiment, was exactly proportional to the stress. The diameter of each of the wires was very nearly 1 mm., and in only one instance* was the value of n carried beyond 200°. As far as could be ascertained the torsional stress never exceeded the limit of elasticity, the recovery being in all instances apparently perfect.†

As soon as the determination of the modulus of torsional elasticity by the statical method had been satisfactorily concluded, the modulus was redetermined by the method of torsional vibrations. The time of vibration was in the case of each wire taken from the mean of a large number of observations, first with only the graduated plate attached to the wire, and again when the moment of inertia of the plate had been supplemented with a hollow ring of copper, turned true inside and outside, and of which the moment of inertia could be calculated with considerable accuracy. The error likely to arise in the determination of the modulus of torsional elasticity by the kinetical method would not in any case be greater than 0·1 per cent. In the next table will be found the results obtained by both methods :—

* That of platinum in which the value of n was 210.

† This does not necessarily imply perfect elasticity, as for this the recovery of the wire on the removal of the stress should be *instantaneous*. Whether this was so could not, of course, be ascertained.

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Fig 1.

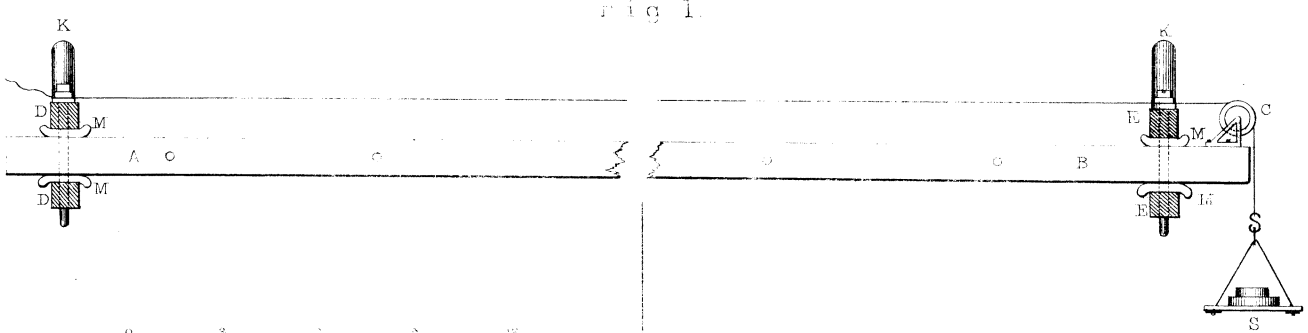
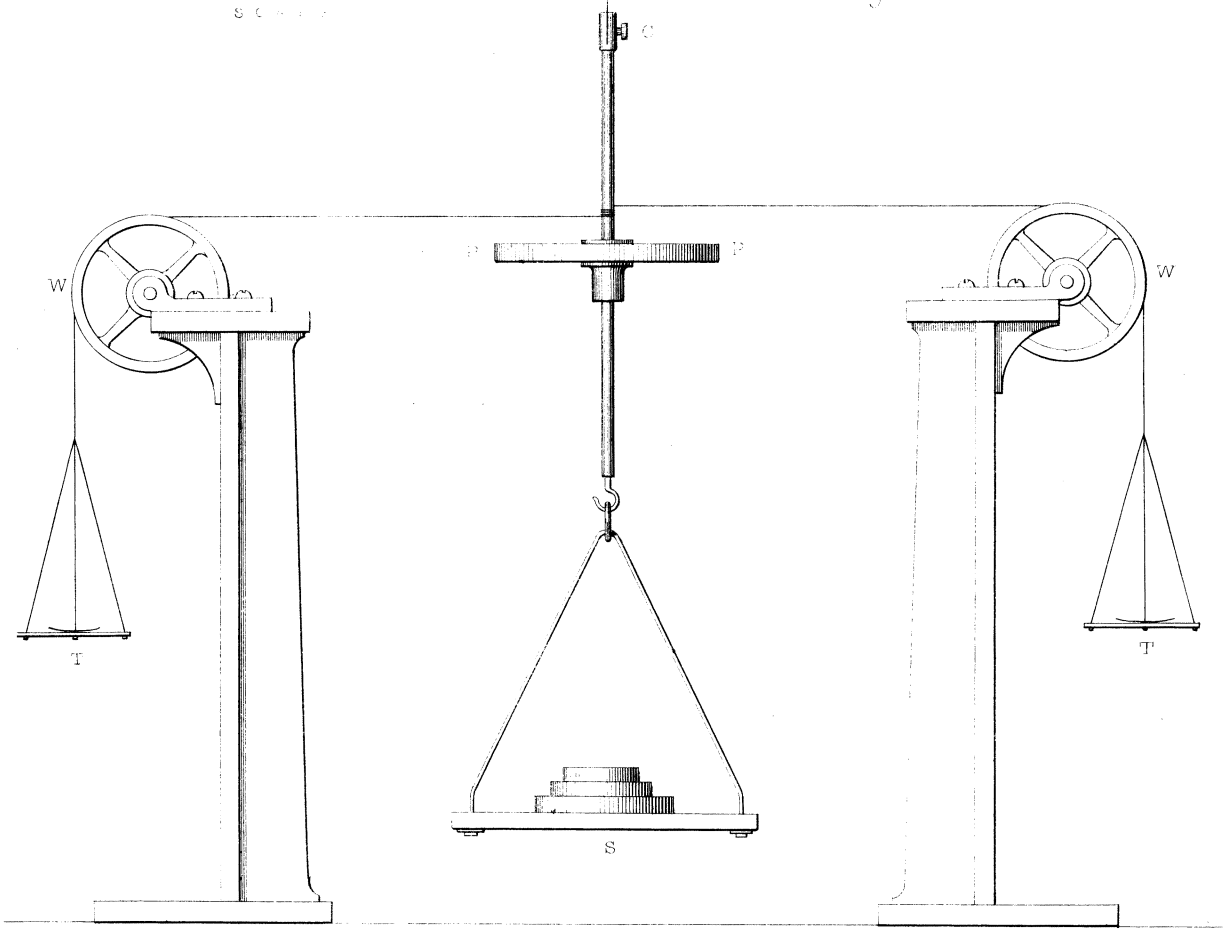


Fig 2.



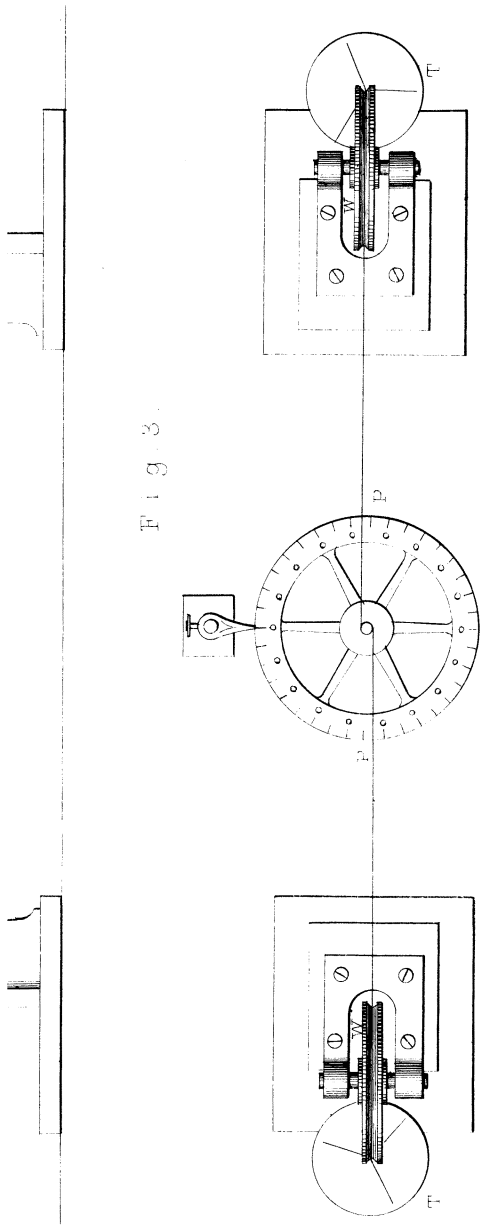


Fig 3.

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Table IV.

Metal.	Condition.	Modulus of torsional elasticity in grams per square centimetre obtained by the statical method. r_s.	Ditto obtained by the kinetical method. r_k.	$\frac{r_s}{r_k}$.
Iron.....	Annealed	$751{\cdot}5 \times 10^6$	$766{\cdot}5 \times 10^6$	1·020
Platinum ...	Unannealed	662·2	663·5	1·002
Silver.....	„	275·5	278·0	1·009
Aluminium..	„	267·7	266·9	0·997

The values of r_k given in Table IV were obtained from the formula—

$$r_k = \frac{2LM\pi^3}{981{\cdot}4t^2s^2},$$

where L is the length of the wire, t the time of vibration, s the section, M the moment of inertia, and 981·4 is the value of g at the place, the units being throughout C.G.S.

It will be seen that for the hard-drawn metals the values of r_s and r_k agree with each other within the limits of errors of observation, and that for these metals the mean value of r_s/r_k is 1·0043. It is impossible in this case to make an exact comparison of the values of r_s and r_k, when for the former allowance is made for the effects of loss and gain of heat, since the times of vibration in the kinetical method were too long* to avoid gain and loss of heat in using the method, but if the correction could be accurately applied, it would evidently on the whole bring still greater accordance between the values of r_k and r_s.

For the annealed iron the value of r_s exceeds that of r_k by an amount which is greater than can be attributed either to heating and cooling effects or to errors of observation.

Summary.

1. The value of the modulus of longitudinal elasticity for hard-drawn metals, as determined by the statical method of loading, accords with the value obtained by the method of longitudinal vibrations, provided the deformations produced in using the former method are sufficiently small.

* The times of vibration varied from 6 to 9 seconds.

2. The velocity of sound in a wire is independent of the load on the wire.

3. The velocity of sound in a wire is not sensibly altered by permanent extensions of the wire, provided sufficient rest be allowed after the permanent extension has taken place.

4. The value of the modulus of torsional elasticity as determined by the statical method, accords with the value obtained by the method of torsional vibrations for most metals in the hard-drawn condition, provided the deformations produced are sufficiently small.

"On the present Position of the Question of the Sources of the Nitrogen of Vegetation, with some new Results, and preliminary Notice of new Lines of Investigation." By Sir J. B. Lawes, Bart., LL.D., F.R.S., and J. H. Gilbert, LL.D., F.R.S. Preliminary Notice.* Received and read June 16, 1887.

For many years past the question of the sources of the nitrogen of our crops has been the subject of much experimental enquiry both at Rothamsted and elsewhere. Until quite recently, the controversy has chiefly been as to whether plants directly assimilate the free nitrogen of the atmosphere; but, during the last few years, the discussion has assumed a somewhat different aspect. The question still is whether the free nitrogen of the air is an important source of the nitrogen of vegetation; but whilst few now adhere to the view that chlorophyllous plants directly assimilate free nitrogen, it is nevertheless assumed to be brought under contribution in various ways, coming into combination within the soil, under the influence of electricity, or of micro-organisms, or of other low forms which thus indirectly serve as an important source of the nitrogen of plants of a higher order. Several of the more important of the investigations in the lines here indicated seem to have been instigated by the assumption that natural compensation must be found for the losses of combined nitrogen which the soil sustains by the removal of crops, and for those which result from the liberation of nitrogen from its combinations under various circumstances.

We propose to summarise some of our own more recently published results bearing on various aspects of the subject, to put on record additional results, to give a preliminary notice of new lines of enquiry, and to discuss the evidence so adduced with reference to the results

* This Preliminary Notice was originally intended to have served as the Abstract of a fuller paper, and is so referred to in the account of the meeting of June 16 (vol. 42, p. 483).

Fig. 1.

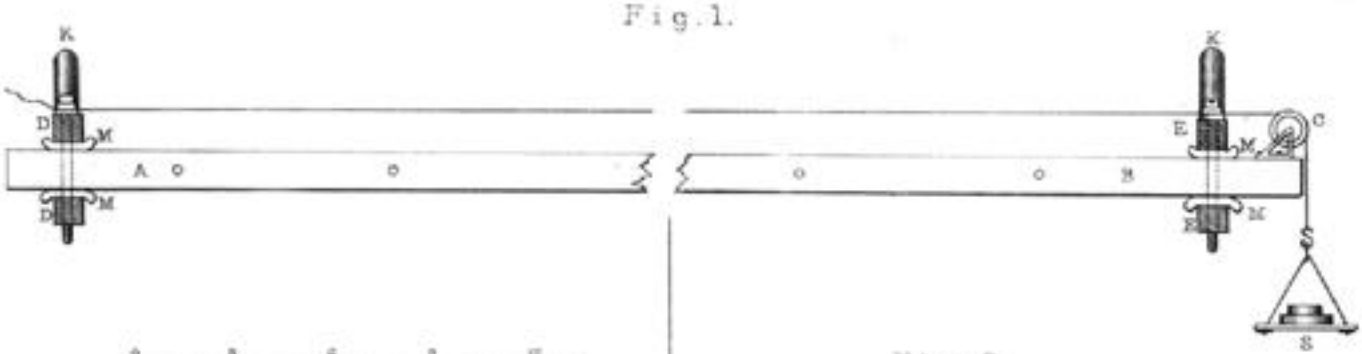


Fig. 2.

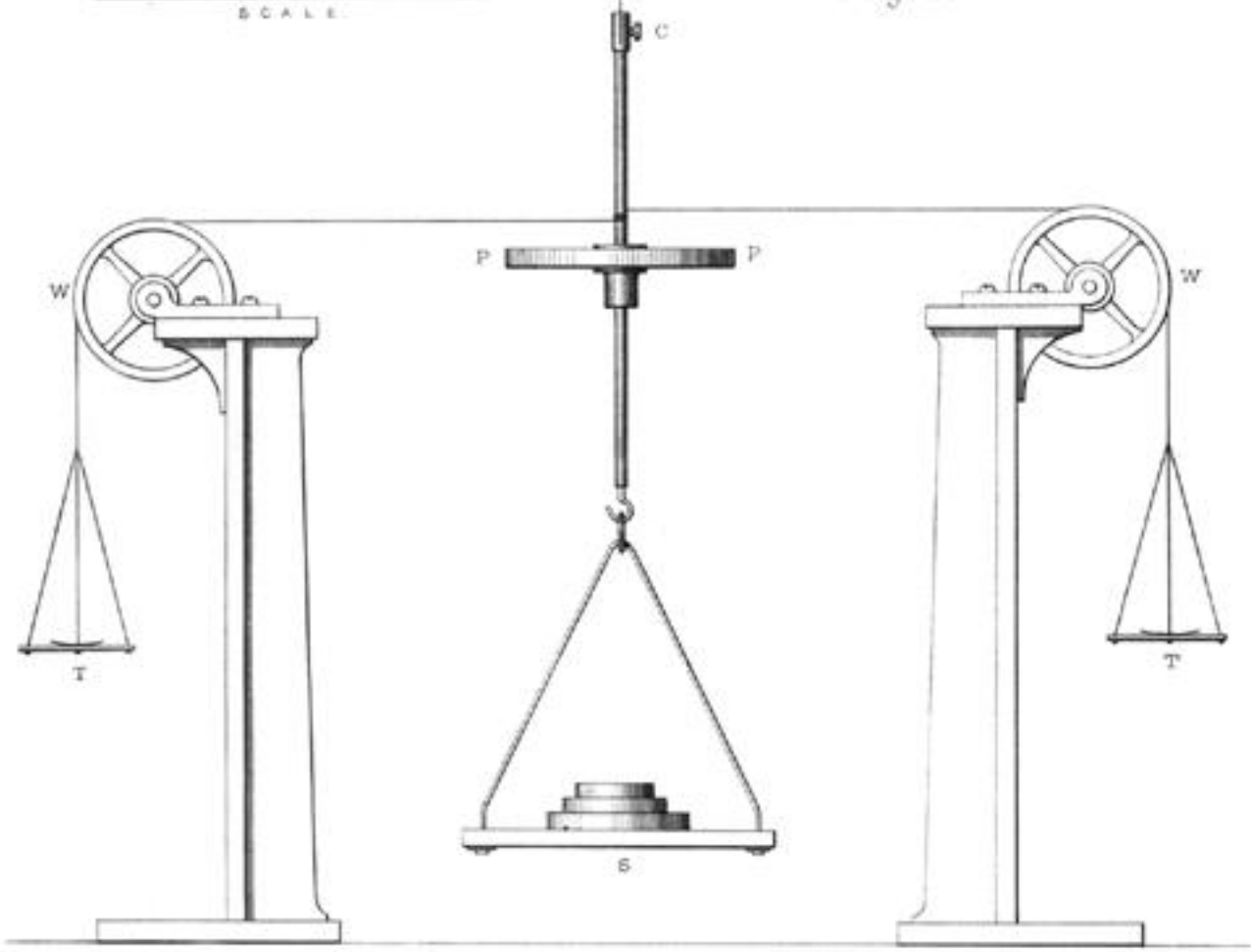
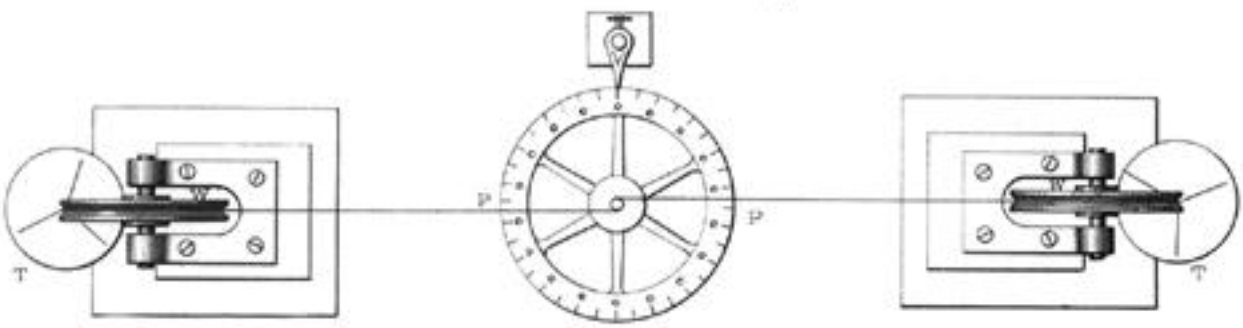


Fig. 3.



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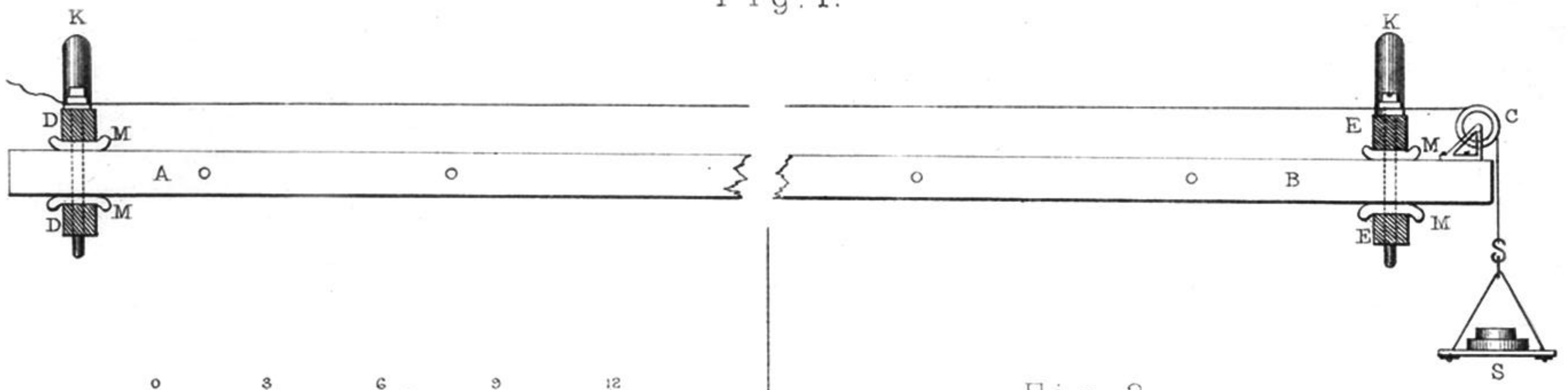

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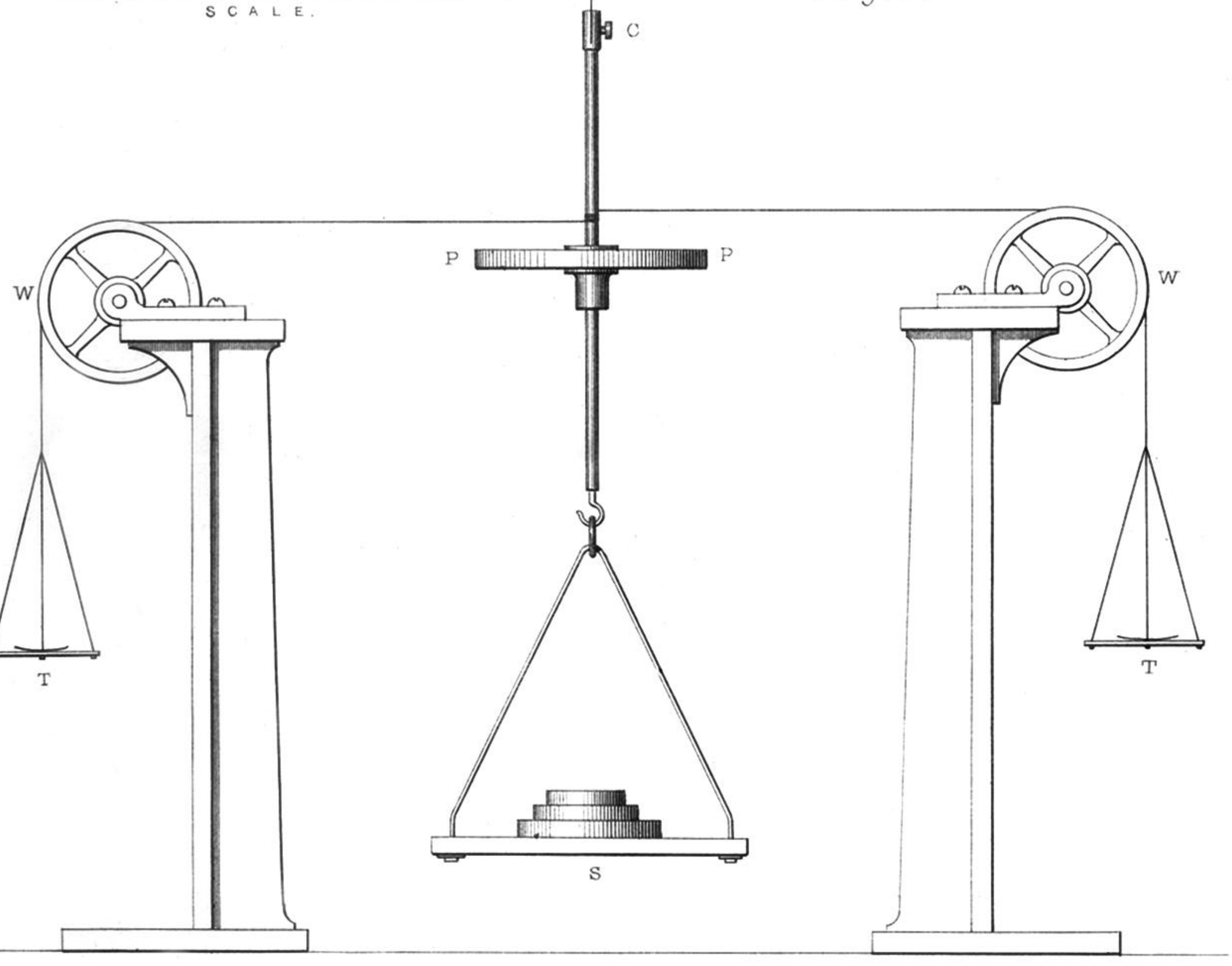

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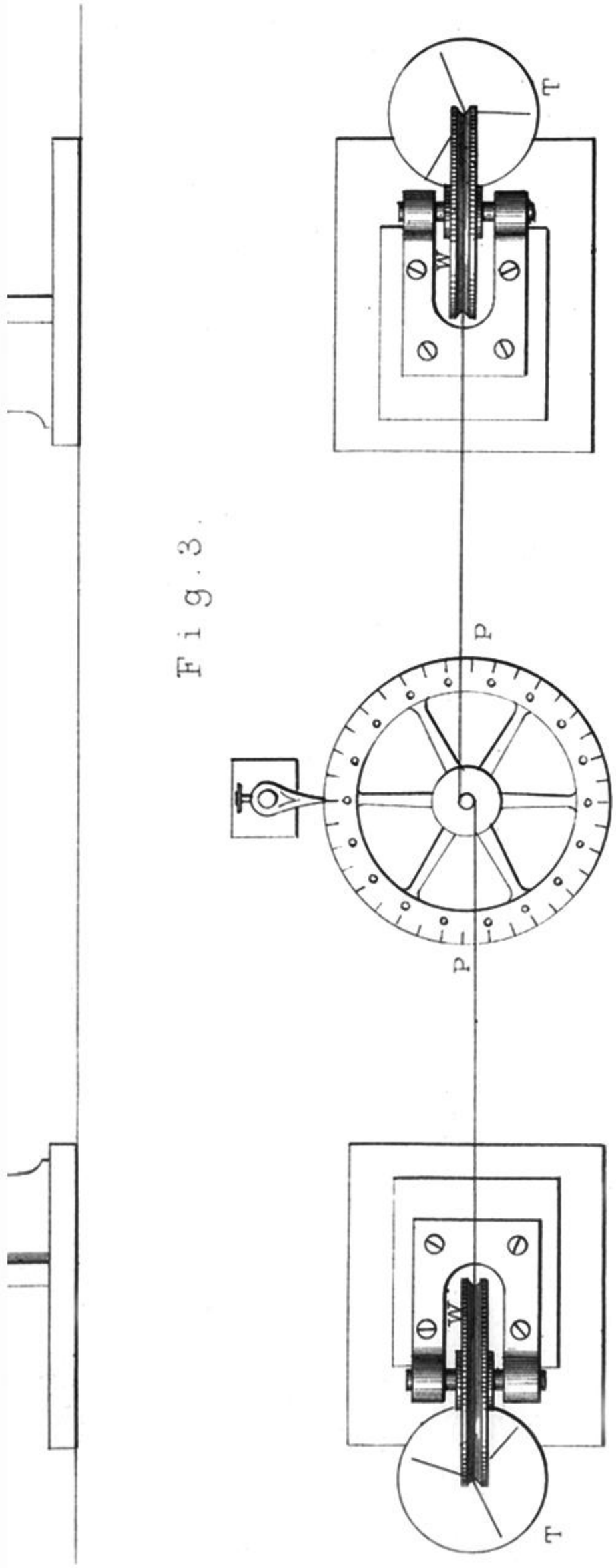

Fig. 3.

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